THEORY OF THE ARCH APPLIED.*

It is hoped that the description and examples given last week, are in sufficient detail to render the use of the table apparent; in the following examples results only will be given. In the three first, arches of the dimensions of those of three of the boldest bridges which have ever been constructed are compared with regard to the stress upon their materials. The depths of the permanent loads and allowance loads over the crowns are taken respectively, at 2 feet 6 inches, and 3 feet, in all of them. The arches are supposed to be so proportioned as not to be stronger at the crowns than at any other parts.

The dimensions of the centre arch of the bridge of La Santa Trinita, at Florence, built by Ammanati are, span, 96 feet; rise, 16 feet; height of keystone, 3·17 feet. Its form, though slightly pointed, as it consists of two intersecting parabolic arcs, does not vary greatly (according to Mr. Cresy, from whose Encyclopædia the above and following dimensions have been copied), from a semi-ellipse. Supposing it then to be elliptical (to avoid broaching the proper formula, however simple it may be). By formula (4), if $T = 3{\cdot}17 + 2{\cdot}5 + 3 = 9{\cdot}67$, $S = 896{\cdot}25$ $C = \frac{896{\cdot}25}{3{\cdot}17} = 279{\cdot}5$. The material is white marble, the modulus of fracture, 5,058 feet, and $\frac{5058}{279{\cdot}5} = 18$, so that the actual pressure upon the materials of such an arch so loaded would be only an eighteenth part of what would crush it, and there would be an excess of resistance to crushing equal to seventeen times it own pressure.

The bridge built at Maidenhead by Mr. Brunel, is semi-elliptical; its arches have a span of 128 feet, rise of 24·25, and depth of arch at the crown of 5·25 feet. If $T = 5{\cdot}25 + 5{\cdot}5 = 10{\cdot}75$, by formula (4) $C = 246{\cdot}7$. The arches are of brick, the modulus of fracture of which material has been estimated at 858 feet, $\frac{858}{246{\cdot}7} = 3{\cdot}45$, and 2·45 is the ratio of the excess of strength.

Chester-bridge, built by Mr. Harrison, has a segmental arch of about 90 degrees, the radius being 141·4 feet, and the span 200 feet: the key-stone is 4 feet deep. If $T = 4 + 5{\cdot}5 = 9{\cdot}5$, and $F = 28$ feet (fig. 3), by formula (3) $C = 385{\cdot}7$ feet. If 5,058 feet be taken for the modulus of fracture, as was done with regard to Ammanatis' bridge, $\frac{5058}{385{\cdot}7} = 13{\cdot}1$, and the excess of its resistance is as 12 to 17 as compared with La Santa Tritita-bridge, and as 12 to 2¼ as compared with Maidenhead-bridge. If 9,551 feet, the reputed modulus of granite be taken, $\frac{9551}{385{\cdot}7} = 24{\cdot}8$, and the comparative numbers are as 24 to 17, and 24 to 2·5. Notwithstanding the boldness of the Maidenhead-bridge, its strength would appear to be excessive if it were to be compared with some structures which depend for strength upon the tenacity of cast-iron.

An arch between the towers of Lincoln Cathedral has been considered a prodigy: its span is 28 feet, versed-sine 9 inches, depth at the crown 11 inches, and breadth 1 foot 9 inches: the linear measure of the horizontal stress is only about 100 feet. This, and such-like arches, from their want of mass and liability to great vibration, are quite unfitted to resist the action of any considerable impact.

It will be useful to consider the case of an arch with equal loads upon the haunches, and with the curve of equilibrium symmetrical. Edwards, a Welch mason, who obtained great reputation as a bridge builder through dearly-bought experience, having built a bridge of three arches over the Taafe, in South Wales, which was destroyed by a flood, was bound by his contract to reinstate it. In order to avoid a similar accident, he determined to span the river with a single arch out of reach of the waters. The arch which was chosen by him was a segment of a circle of about 100 degrees, the chord being 140 feet, the versed-sine [illegible] feet, and the radius about 87·6 feet. The [illegible] of the arch was 2 feet 6 inches. [illegible] bridge was complete, with the exception [illegible] parapets, it failed, and the unfortunate mason had to commence his work a third time. He perceived that he had overloaded the haunches, which being flattened and descending, gave a greater curvature to the crown, and caused it to ascend. To remedy this defect, not varying the form of the intrados, he constructed the spandrils of his third bridge with hollow spaces in the shape of barrels, and was at length successful. Had this intelligent practical man possessed the knowledge of a Rennie or a Southern, and examined his second design by the principles of equilibrium, he no doubt would have discovered in what respect it was faulty.

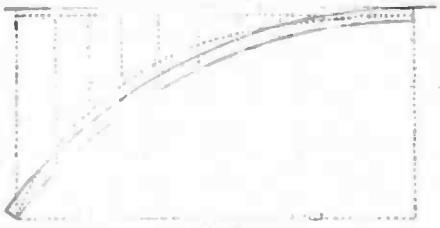

Fig. 6.

Let an arch of the above dimensions (fig. 6) be supposed backed up to the level of the top of the key-stone. By the first formula, the horizontal stress is as 509·42 feet, its linear measure is 203·7 feet; a pressure such as this, though not sufficient to crush, would, in all probability, compress the materials; it is important, therefore, that it should be distributed equally (for if one side of an arch be compressed the opposite side will be relieved from pressure, and the joints perhaps open), and this would be the case if the curve of equilibrium were included within the thickness of the arch, though the less it approaches the extrados the better. It should be mentioned that no account is ever taken of the slight degree of cohesion which exists by means of the cement between the beds of the voussoirs; Edwards might, perhaps, have prevented his arch from springing, for a time, had he tied the key to the springing stones with strong iron bars or by cramping the joints where likely to open—if he, a good practical mason, could ever have thought of so miserable a substitute for good construction.

The ordinates of the equilibral curve, from 5 to 45 degrees, calculated by formula (7), are ·22, ·62, 1·51, 2·97, 5·14, 8·32, 12·36, 17·61, and 23·96 feet, and are so laid down in the figure, which shews that the thickness of 2 feet 6 inches given to the voussoirs would not be sufficient to include the curve if the extradossing had been carried up regularly to the level supposed. The variation is so great, that it is difficult to conceive how such a bridge could have been constructed nearly to completion (notwithstanding the support of a tolerable centre), without recourse being made to the expedient of loading over the crown as the lower part of the extradossing was proceeded with to preserve, in some degree, the equilibrium, and it may be supposed that when this resource failed, with it the bridge failed. Had Edwards increased the thickness of the voussoirs towards the abutments, in the manner now generally practised, there is no doubt that his second bridge would have stood, notwithstanding the injudicious loading over the haunches and the excessive lightness at the crown. In this case there are five points of fracture, one at each abutment, where the curve of equilibrium intersects the intrados, one over each haunch at the point of the extrados, most distant from the curve of equilibrium, and one at the underside of the vertex of the arch. It is at these points that fracture would be *likely* to occur; they may be defined as points of partial or greatest pressure; for it does not necessarily follow, that such an arch so loaded would be broken or even change form: this depends upon the resistance of the materials near those parts, but they would be more strained than those on the opposite side of the thickness of the arch. The curve of equilibrium has been sometimes denominated the line of resistance; it is evident from the way in which it is determined (from the pressure), that it is not related, but rather opposed to the resistance of the materials; it is sometimes, and with propriety, called the line of pressure.

Before taking leave of the subject, a few words with regard to abutment walls, upon whose stability the stability of arches must depend, will not be out of place. Two of the methods which have been employed to determine the thickness of their bases will be adverted to. It is assumed in the following method that an abutment is a perfectly solid mass, whose outer edge (*s*), fig. 1, is a fulcrum, about which the horizontal thrust would cause the abutment to be turned, and to be lifted from its foundation, if the momentum of the thrust were to exceed the momentum of the gravity of the abutment (V), and the vertical momentum of the arch, and load upon it (W). The least thickness, therefore, which can be given to the base, upon this supposition, is where the $(st) \times w + (ru) \times V = S \times (ot)$. If the wall be rectangular, as represented in the figure, $tq = 40$ feet, and $S = 201{\cdot}375$, the thickness (*st*) determined by the above equation is about 8 feet 3 inches; with this thickness only the wall would be in the same condition of equilibrium as a mass (*qrsv*), so overhanging its base that a perpendicular line from its centre of gravity would coincide with one of its edges (*s*). The builder, if he choose this mode of calculation, must be mindful of this circumstance, and will do well to double (at least) in practice, the theoretical base.

It is now usual with large arches to incline the joints of the abutment stones like the joints of an arch, and the second method adverted to rather squares with this excellent practice, as it consists in continuing the curve of equilibrium through the abutments to the foundation, account only being taken of a portion of the materials intended to be eventually employed, and as though that portion were a continuation of the arch. If a builder choose this method, he may find any number of ordinates upon the principles before explained.

This method is illustrated in Fig. 4, the only part taken into account in calculating the ordinates (*mn*), (*op*), (*qr*), is that in the form of a flying buttress between (*i*) and (*q*) 6 feet deep; they were taken at 10 feet apart, and were found to be respectively 16·4, 25, and 34·9 feet.

The curve of equilibrium being traced to the foundation, the question arises as to the ability of the foundation to resist the pressures to which it may be subject, a matter rather for the exercise of a builder's practical knowledge, in each particular case, than of calculation.

The principles applied above to the arch are of universal application, and have long been familiar to the well-instructed architect and engineer. The artisan who has to construct with the work of his hands the designs of others, should also be acquainted with them to enable him to do his office with security to himself; and they ought to be a standing subject for illustration at all mechanical institutions intended for his benefit. The principle of equilibrium in relation to the resistance of materials is due to the great Galileo, who, in his work* upon the subject published in 163-, designates his invention as a new science. The doctrine of equilibrium, relating to weight only, is as old as the time of Aristotle; with this, the architects of the middle ages, if we may judge from their works, in which a much nearer approach is to be observed to an exact equipoise of the masses than in any ancient or modern structures, appear to have been thoroughly acquainted. As to the resistance of the materials introduced into the estimate by Galileo, they would not be much behind us in knowledge; for it must be confessed that our knowledge at present of the resisting qualities of stones, &c., derived from experiments, is very imperfect, and that the numbers, measures, or moduli, as they are called, derived from them, must be considered rather as relative than positive.

J. P. W.

See p. 295, *ante*.

* The principles explained in the portion of this remarkable work, entitled, "De Motu Locali," have produced most ample results under the treatment of his successor, Newton, and Newton's followers. That Galileo was perfectly aware of the importance of his inventions is evident from the following prophetical passage, as well as from a parallel passage in the printer's preface:—"Observatum est, nimirum, seu projecta, lineam qualitercunque curvam describere; veruntamen eam esse parabolam nemo prodidit. Hæc ita esse, et alia non pauca, nec minus scitu digna, a me demonstrabuntur: et quod pluris faciendum facile, additus et accessus ad amplissimam, admirantibusque scientiam, cujus hi nostri labores erunt elementa, recludet: in qua ingenia meo perspicaciora abditiores recessus penetrabunt."

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